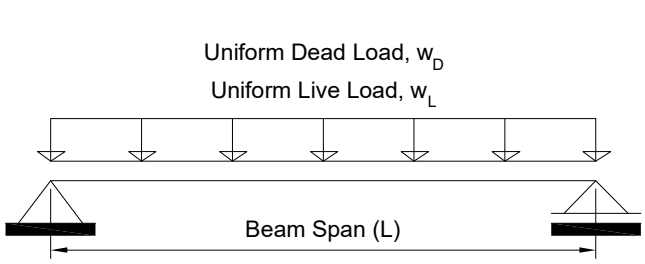
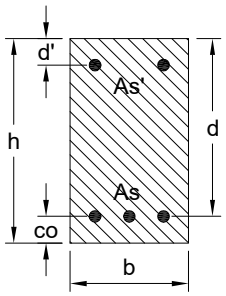
Calculation of Deflection for Simple Support Concrete Beam Under Uniform Loads

As per ACI 318-11 Chapter 9





System

Width of Concrete Section, b=			12.0 in
Depth of Concrete Section, h=			22.0 in
Concrete Cover, co=			2.5 in
Effective Depth of Concrete Section, d=	h - co = 22.0 - 2.5	=	19.5 in
Depth of Compression Reinforcement, d'=			2.5 in
Area of Tension Reinforcement, A_s=			1.80 in^2
Area of Compression Reinforcement, A_s'=			0.6 in^2
ρ=	As / (b * d)	=	0.0077
ρ'=	As' / (b * d)	=	0.0026
Distance from Centroidal Axis of Gross Section, y_t= h / 2		=	11.0 in
Beam Span, L=			25.0 ft

Load

Uniform Dead Load, w_D=			0.395 kip/ft
Uniform Live Load, w_L=			0.300 kip/ft
Percentage of Sustained Live Load, Sus=			50 %
Moment due to Dead Load, M_D=	$w_D * L^2 / 8$	=	30.9 kip*ft
Moment due to Live Load, M_L=	$w_L * L^2 / 8$	=	23.4 kip*ft
Sustained Moment, M_{sus}=	$M_D + (Sus/100)*M_L$	=	42.6 kip*ft

Material Properties

Concrete Strength, f'$_c$=	3000 psi
Yield Strength of Reinforcement, f_y=	40000 psi
Modulus of Elasticity of Reinforcement, E_s=	29000000 psi
Modification Factor for Lightweight Concrete, λ=	1.00
Concrete Density, w_c=	150 psi

Properties of Cracked Section

Modulus of Rupture (According to Eq. 9-10 of ACI318), $f_r = 7.5*\lambda*\sqrt{f'_c}$	=	411 psi

Modulus of Elasticity of Concrete (According to Cl. 8.5.1 of ACI318),

$E_c =$ $w_c^{1.5} * 33 * \sqrt{f'_c}$ = 3320561 psi

$n_s = E_s / E_c$ = 8.7

$I_g =$ $b * h^3 / 12$ = 10648 in^4

$B =$ $b / (n_s * A_s)$ = 0.77 in

$r =$ $\dfrac{(n_s - 1) * As'}{n_s * A_s}$ = 0.295

$kd =$ $\dfrac{\sqrt{2 * d * B * \left(1 + r * \frac{d'}{d}\right) + (1 + r)^2} - (1 + r)}{B}$ = 5.76 in

$I_{cr} =$ $\dfrac{b * kd^3}{3} + n_s * As * (d - kd)^2 + (n_s - 1) * As' * (kd - d')^2$ = 3770 in^4

Cracking Moment (According to Eq. 9-9 of ACI318),

$M_{cr} =$ $f_r * I_g / (y_t * 12000)$ = 33.2 kip*ft

Properties of Effective Section

Effective Moment of Inertia (According to Eq 9-8 of ACI318):

$I_{e\_Dead1} =$ $\left(\dfrac{M_{cr}}{M_D}\right)^3 * I_g + \left(1 - \left(\dfrac{M_{cr}}{M_D}\right)^3\right) * I_{cr}$ = 12301 in^4

$I_{e\_Dead} =$ $MIN(I_g ; I_{e\_Dead1})$ = 10648 in^4

$I_{e\_Sus1} =$ $\left(\dfrac{M_{cr}}{M_{sus}}\right)^3 * I_g + \left(1 - \left(\dfrac{M_{cr}}{M_{sus}}\right)^3\right) * I_{cr}$ = 7026 in^4

$I_{e\_Sus} =$ $MIN(I_g ; I_{e\_Sus1})$ = 7026 in^4

$I_{e\_All1} =$ $\left(\dfrac{M_{cr}}{M_D + M_L}\right)^3 * I_g + \left(1 - \left(\dfrac{M_{cr}}{M_D + M_L}\right)^3\right) * I_{cr}$ = 5342 in^4

$I_{e\_All} =$ $MIN(I_g ; I_{e\_All1})$ = 5342 in^4

Short Term Deflection

$\Delta_{i\_Dead} =$ $\dfrac{5 * M_D * L^2 * 12^3}{48 * E_c * I_{e\_Dead} / 1000}$ = 0.098 in

$\Delta_{i\_Sus} =$ $\dfrac{5 * M_{sus} * L^2 * 12^3}{48 * E_c * I_{e\_Sus} / 1000}$ = 0.205 in

$\Delta_{i\_All} =$ $\dfrac{5 * (M_D + M_L) * L^2 * 12^3}{48 * E_c * I_{e\_All} / 1000}$ = 0.344 in

Δ_{i_Live}= Δ_{i_All} - Δ_{i_Dead} = 0.246 in

Long Term Deflection

Duration of Sustained loads, Dur: SEL("ACI/Sustained";Dur;) = 5 Years or more

Time-Dependent Factor for Sustained Loads (According to Cl. 9.5.2.5 of ACI318):

ξ= TAB("ACI/Sustained";x;Dur=Dur;) = 2.00

Multiplier Factor for Long-Term Deflection (According to Eq. 9-11 of ACI318),

λ_Δ= ξ/ (1 + 50*ρ') = 1.77

Creep and Shrinkage Deflection, Δ_{cp_sh} = λ_Δ * Δ_{i_Sus} = 0.36 in

Δ_{total}= Δ_{cp_sh} + Δ_{i_Live} = 0.61 in

Calculation Summary

Long Term Deflection, Δ_{total}= Δ_{cp_sh} + Δ_{i_Live} = 0.61 in